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General form of a factorable polynomial

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Abstract :
This paper provides extension to the types of factoring of polynomials to illustrate the fact that multiplication with polynomials has no restrictions when it comes to the number of terms so does factoring. It aims to develop students' perception about polynomials and to motivate them to continue discovering factorable polynomials.that are new to us.The idea originated from the expression $x^4 + x^2 y^2 + y^4$ which is a product of $x^2 + xy + y^2$ and $x^2 - xy + y^2$. Factor theorem, rational zeros of a polynomial, and symmetric and transitive properties of real numbers were used in this study in proving other factorable polynomials. The contents include the difference of perfect nth powers $(ax)^n - (by)^n$, the trinomial of the form $x^n + (xy)^{n/2} + y^n$, and the general form of a factorable polynomial $p(x) = x^n + x^{n-d} y^d + x^{n-2d} y^{2d} + \ldots + x^{2d}y^{n-2d} + x^d y^{n-d} + y^n$ for some positive integers n and d with a and b as real numbers.

Keywords :
Mathematics, factorable polynomials, student perceptions
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Introduction

Are the types of factorable polynomials introduced in Algebra books the only factorable algebraic expressions? This question confused the researcher when she solved $x^6 - y^6$ in difference of two squares and difference of two cubes because the results were $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$ and $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$. In other words, some polynomials are taught to be not factorable because its form is not in the six types of polynomials.This study was investigated with the aim of finding other trinomials and other expressions that are factorable. It is also anticipated to give opportunity to readers to investigate other factorable polynomials.

Statement of the problem

This study was made in order identify some factorable polynomials. Specifically, it sought to answer the following questions.

1. What are the other factorable binomials?
2. What are the other factorable trinomials?
3. What is the general form of polynomials?

Significance of the study

It is anticipated that the outcomes of this study will provide insights and guidelines to students, teachers, and educators concerning the discovery of other factorable polynomials. The students will eventually know that there are other polynomials which can still be factored. They must not hesitate to find other algebraic expressions that can be expressed as products of other expressions.

The teachers must incorporate the results in this present study and convince other students to discover more polynomials similar to this concept.The educators must also incorporate the results in this present study in the existing mathematics curriculum to include it in the topics taught by mathematics teachers.

Finally, the result in this study will serve as basis for the educational reform of mathematics lessons with the involvement of real – life application of the mathematics different concepts.

Scope and Delimitation of the Study

This study investigated the other factorable polynomials. It looked on the trinomials, binomials, and general form of polynomials.

Theoretical Framework

Factor theorem, difference of two squares, transitive property of equality, and the general form of a polynomial were used in this study.

By factor theorem, if P(x) is a polynomial and r is any real number then $x - r$ is a factor of P(x) if and only if $P(r) = 0$. This means that for any n is a positive integer,
$x^n - y^n$has a factor of $x - y$ and it is used in this present study.

Using transitive property of equality,$(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2) = (x - y)(x + y)(x^4 + x^2y^2 + y^4)$ is also equals to $x^6 - y^6$. This was the concept that the researcher started to investigate other trinomials that are factorable.

Now, the general form of a polynomial is $P(x) = x^n + x^{n-1} + x^{n-2} + \ldots + x + 1$. In this idea, the researcher thought of a possible standard form of a factorable polynomial until she had formulated it in this present study.

Conceptual Framework

The six types of factoring that we knew were just the reverse of special products, which are limited to three terms. Polynomials containing four or more than four can only be possible factored using grouping of terms, in such a way that the other types of factoring can be used. This idea is what we believed, but we forget that multiplication has no restrictions to the number of the terms of expressions, and that factoring is a reverse of multiplication. For instance, expressions like $x^6 – y^6$ can be factored in two ways; that is, using difference of perfect squares or difference of perfect cubes. However, factor theorem gives us the answer that $x^6 – y^6 = (x – y)\ (x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5)$ and by comparison from the results of using difference of two squares and difference of two cubes, $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 – xy + y^2)$. So, by transitive property, $x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5 = (x + y)(x^2 + xy + y^2)(x^2 – xy + y^2)$.

The result of transitivity of some factors of $x^6 – y^6$ convinced the researcher to investigate other terms that might be factorable which leads to three theorems of this present study.

Corollary 1 Difference of Perfect nth Powers

Let a, b, and n be positive integers. Then, $(ax)^n – (by)^n = (ax – by)[(ax)^{n-1} + (ax)^{n-2}(by) + (ax)^{n-3}(by)^2 + (ax)^{n-4}(by)^3 + \ldots + (ax)^3(by)^{n-4} + (ax)^2(by)^{n-3} + (ax)(by)^{n-2} + (by)^{n-1}]$.

Proof: (⇨) Let $P(x) = (ax)^n – (by)^n$. Suppose that $ax – by$ is not a factor of P(x). Then for $x = (by)/a$, $P[(by)/a] \neq 0$.By direct substitution of the value of x to P(x),

$P(x) = (ax)^n – (by)^n$

$P[(by)/a] = [(a)(by)/a]^n – (by)^n$

$P[(by)/a] = (by)^n - (by)^n$

$P[(by)/a] = 0$.

This is a contradiction to our assumption. Thus, $ax – by$ is a factor of P(x) by factor theorem.

(⇦)Multiplying $(ax – by)$ and $[(ax)^{n-1} + (ax)^{n-2}(by) + (ax)^{n-3}(by)^2 + (ax)^{n-4}(by)^3 + \ldots + (ax)^3(by)^{n-4} + (ax)^2(by)^{n-3} + (ax)(by)^{n-2} + (by)^{n-1}]$, we get $(ax)^n – (by)^n$.

Examples:

1. $16p^4 – 81r^4 = (2p – 3r)(8p^3 + 12p^2r + 18pr^2 + 27r^3)$
2. $x^6 – y^6 = (x – y)(x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5)$

Theorem 2 Trinomial of the Form $x^n + (xy)^{n/2} + y^n$

The trinomial, $x^n + (xy)^{n/2} + y^n$is factorable into $[x^n + (xy)^{n/2} + y^n]$ and $[x^n + (xy)^{n/2} + y^n]$ for any nonzero integer n.

Proof: (⇨) Adding + +] – + +] to

$x^n + (xy)^{n/2} + y^n$, we have:

$[\ x^n + (xy)^{n/2} + y^n\ ] + \ + \ + \ ] – \ + \ + \ ]$

$= \ x^n + (xy)^{n/2} + y^n \ + \ + \ + \ – – \ –$

$= x^n – \ + (xy)^{n/2} \ + – \ + \ + – \ + y^n$

$= [\ x^n – \ + (xy)^{n/2}\ ] + [– \ + \ ] + [\ – \ + y^n\ ] = \ – \ + \ [– \ –$

$= [–$

(⇦) The product of [– equals to$x^n+(xy)^{n/2} + y^n$.

Corollary 3. If n is a non-zero real number then $x^n + (xy)^{n/2} + y^n = [x^n + (xy)^{n/2} + y^n]\ [x^n + (xy)^{n/2} + y^n]$.

Examples:

1. $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)\ (x^2 - xy + y^2)$

2. $x^8 + x^4 + 1 = (x^4 + x^2 + 1)\ (x^4 - x^2 + 1) = (x^2 + x + 1)\ (x^2 - x + 1)\ (x^4 - x^2 + 1)$

Theorem 4. General Form of a Factorable Polynomial

Let q, m, n, r, and d be positive integers greater than 1 such that q = mr. Then, if the polynomial $P(x) = x^n + x^{n-d}y^d + x^{n-2d}y^{2d} + x^{n-3d}y^{3d} + \ \dots\ + x^{2d}y^{n-2d} + x^dy^{n-d} + y^n$ has q terms then it is factorable into $x^{n-(m-1)d} + x^{n-(2m-1)d}y^{md} + x^{n-(3m-1)d}y^{2md} + \dots + x^{2md}y^{n-(3m-1)d} + x^{md}y^{n-(2m-1)d} + y^{n-(m-1)d}$ and $x^{(m-1)d} + x^{(m-2)d}y^d + x^{(m-3)d}y^{2d} + \dots + x^{2d}y^{(m-3)d} + x^dy^{(m-2)d} + y^{(m-1)d}$.

Proof: (⇨) Suppose we extend P(x) and re – group them into m terms each then,

$P(x) = x^n + x^{n-d}y^d + x^{n-2d}y^{2d} + x^{n-3d}y^{3d} + \ \dots\ x^{n-(m-2)d}y^{(m-2)d} + x^{n-(m-1)d}y^{(m-1)d} + x^{n-md}y^{md} + x^{n-(m+1)d}y^{(m+1)d} + \dots + x^{n-(2m-2)d}y^{(2m-2)d} + x^{n-(2m-1)d}y^{(2m-1)d} + x^{n-2md}y^{2md} + x^{n-(2m+1)d}y^{(2m+1)d} + \dots + x^{n-(3m-2)2d}y^{(3m-2)d} + x^{n-(3m-1)d}y^{(3m-1)d} + \dots +$
$x^{n-(2m-1)d}y^{(2m-1)d} + x^{n-(2m-2)d}y^{(2m-2)d} + x^{n-(2m-3)d}y^{(2m-3)d} + \dots + x^{(m+1)d}y^{n-(m+1)d} + x^{md}y^{n-md} + x^{(m-1)d}y^{n-(m-1)d} + x^{(m-2)d}y^{n-(m-2)d} + \dots + x^{2d}y^{n-2d} + x^dy^{n-d} + y^n$
$= [x^n + x^{n-d}y^d + x^{n-2d}y^{2d} + x^{n-3d}y^{3d} + \ \dots\ x^{n-(m-2)d}y^{(m-2)d} + x^{n-(m-1)d}y^{(m-1)d}] + [x^{n-md}y^{md} + x^{n-(m+1)d}y^{(m+1)d} + \dots + x^{n-(2m-2)d}y^{(2m-2)d} + x^{n-(2m-1)d}y^{(2m-1)d}] +$
$[x^{n-2md}y^{2md} + x^{n-(2m+1)d}y^{(2m+1)d} + \dots + x^{n-(3m-2)2d}y^{(3m-2)d} + x^{n-(3m-1)d}y^{(3m-1)d}] + \dots +$
$[x^{n-(2m-1)d}y^{(2m-1)d} + x^{n-(2m-2)d}y^{(2m-2)d} + x^{n-(2m-3)d}y^{(2m-3)d} + \dots + x^{(m+1)d}y^{n-(m+1)d} + x^{md}y^{n-md}] + [x^{(m-1)d}y^{n-(m-1)d} + x^{(m-2)d}y^{n-(m-2)d} + \dots + x^{2d}y^{n-2d} + x^dy^{n-d} + y^n]$
$= x^{n-(m-1)d}y^{(m-1)d}[x^{(m-1)d} + x^{(m-2)d}y^d + x^{(m-3)d}y^{2d} + x^{(m-4)d}y^{3d} \dots + x^dy^{(m-2)d} + y^{(m-1)d}] + x^{n-(2m-1)d}y^{md}[x^{(m-1)d} + x^{(m-2)d}y^d + x^{(m-3)d}y^{2d} + x^{(m-4)d}y^{3d} \dots + x^dy^{(m-2)d} + y^{(m-1)d}] + x^{n-(3m-1)d}y^{2md}[x^{(m-1)d} + x^{(m-2)d}y^d + x^{(m-3)d}y^{2d} + x^{(m-4)d}y^{3d} \dots + x^dy^{(m-2)d} + y^{(m-1)d}] + \dots + x^{md}y^{n-(2m-1)d}[x^{(m-1)d} + x^{(m-2)d}y^d + x^{(m-3)d}y^{2d} + x^{(m-4)d}y^{3d} \dots + x^dy^{(m-2)d} + y^{(m-1)d}] + y^{n-(m-1)d}[x^{(m-1)d} + x^{(m-2)d}y^d + x^{(m-3)d}y^{2d} + x^{(m-4)d}y^{3d} \dots + x^dy^{(m-2)} + y^{(m-1)d}]$

$= [x^{n-(m-1)d}y^{(m-1)d} + x^{n-(2m-1)d}y^{md} + x^{n-(3m-1)d}y^{2md} + x^{md}y^{n-(2m-1)d} + y^{n-(m-1)d}]$
$[x^{(m-1)d}y^{n-(m-1)d} + x^{(m-2)d}y^{n-(m-2)d} + \dots + x^{2d}y^{n-2d} + x^dy^{n-d} + y^n]$

(⇦) The product of the two factors equals the polynomial P(x).

Examples:

Polynomial	Factors	Reason
$x^{10} + x^8 + x^6 + x^4 + x^2 + 1$	$(x^8 + x^4 + 1)(x^2 + 1)$	General form of a factorable polynomial (q = 6, m = 2, r = 3, and d = 2)
$(x^8 + x^4 + 1)$	$(x^4 + x^2 + 1)\ (x^4 - x^2 + 1)$	Trinomial of the Form $x^n + (xy)^{n/2} + y^n$
$(x^4 + x^2 + 1)$	$+ 1)(x^2 - x + 1)$	Trinomial of the Form $x^n + (xy)^{n/2} + y^n$
${}^{10} + x^8 + x^6 + x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)(x^4 - x^2 + 1)\ (x^2 + 1)$.		
${}^{21} + x^{18}y^3 + x^{15}y^6 + x^{12}y^9 + x^9y^{12} + x^6y^{15} + x^3y^{18} + y^{21}$	${}^{12})(x^9 + x^6y^3 + x^3y^6 + y^9)$	General Form of a Factorable Polynomial

		(q = 8, m = 4, r = 2, and d = 3)
	$y^4)^3](x^6 + y^6)(x^3 + y^3)$	General Form of a Factorable Polynomial (q= 4, m= 2, r = 2, and d = 3) and Sum of Two Perfect Cubes
	$) (x^8 – x^4y^4 + y^8)$ $(y^2)^3] (x + y)$ $+ y^2)$	Sum of Two Perfect Cubes
	$) (x^8 – x^4y^4 + y^8)$ $) (x^4 - x^2y^2 + y^4)$ $(x^2—xy + y^2)$	Sum of Two Perfect Cubes
The expression, $x^{21} + x^{18}y^3 + x^{15}y^6 + x^{12}y^9 + x^9y^{12} +x^6y^{15} +x^3y^{18} + y^{21}= (x^4 + y^4) (x^8 – x^4y^4 + y^8)(x^2 + y^2) (x^4 - x^2y^2 + y^4)(x + y) (x^2—xy + y^2)$.		

Conclusion

Modern world views how technologies develop our lives today. This continuation of the expansion of technology is possible because of the powers of numbers that manipulate theorems, computer programming, and other inventions. Thus, this study does not restrict for the possibilities of discovering other types of factoring and its further extensions and applications.

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